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Parameter-dependent Input Normalization: Direct-Adaptive control with Uncertain Control Direction

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Abstract— **In this work, we propose a direct-adaptive MRAC for relative-degree-unity SISO systems with unknown control direction. The proposed scheme, employing an original construction of the control law and the use of an adaptive observer, achieves the long-searched objective of injecting, through the input, the unmeasurable derivative of the output error. The output derivative injection is performed by a smart construction of the control input that features a Parameter-dependent Input Normalization (PIN). The PIN scheme does not make use of Nussbaum functions usually invoked in the direct-adaptive setting, does not require persistence of excitation of indirect adaptive schemes, does not require switching between multiple models, does not suffer from singularities and does not require to know a-priori bounds on the norm of the high-frequency gain and on the parameters. Effectiveness of the algorithm is illustrated by a numerical example.**

I. Introduction

As one of the main approaches to adaptive control, Model Reference Adaptive Control (MRAC) has been extensively studied over the half past century, due to its both theoretical and practical significance. Among various approaches developed under MRAC framework (see [1][2] and references therein), direct MRAC scheme has attracted considerable amount of attention since it bypasses the estimation of parameters and states of plant, instead, achieves the asymptotic tracking of desired output signal via direct adaptation of the controller parameters. However, despite the intrinsic simplicity and effectiveness of the idea of direct adaptation, the output-error parameterization essentially leads to a bilinear regression form [3], where the high frequency gain, denoted by b in the sequel, appears multiplying the controller parameters. The vast majority of widely recognized direct MRAC methods, including the celebrated augmented error-based approaches, overcome this bilinear difficulty by assuming the sign of the high frequency gain is known in advance. This assumption severely hampers the implementation of direct MRAC to engineering applications with unknown control direction [4][5] and makes the extension to MIMO system more challenge.

The necessity of prior knowledge of control direction was first questioned by Morse [6], in which he conjectured that no rational controller exists capable of adaptive stabilizing an uncertain system without knowing the sign of b. Breakthrough comes with the Nussbaum's idea of employing an oscillating gain function [7] that changes the estimator vector filed periodically. In spite of many efforts [8][9][10] have been devoted to researching and advancing the class of Nussbaum-type control and adaptive laws, it is argued in [11][12] that this kind of algorithms is only of theoretical interest since the recurrent destabilization adaptation leads to an inferior transient behaviour which is practical inadmissible. Alternative solution to avoid knowing the sign of b is to adopt a switching projection [13]. However, the erratic behaviour brought by possible non-stop switching and the issue of unverifiability PE assumption on the regressor signal has not been solved yet. A dynamic regressor extension and mixing estimators-based (DREM-based) scheme reported in [14] ensures the switching happens at most once via exploiting the monotonicity of the parameter estimation error. Nevertheless, the global tracking can only be guaranteed for a sufficiently rich reference signals, which in general unrealistic for a MRAC problem. In this context, we believe, the issue concerning the high-frequency gain assumption made in MRAC remains open and seems far from settled.

In this paper, a classic direct MRAC design problem without the prior knowledge of high frequency gain is addressed for a LTI SISO system with relative degree one. We proposed a novel PIN-based controller along with an original derivative injection gain function that achieves the injection of the derivative of tracking errors via input. In this way, an algebraic linear-in-the parameter error equation is obtained, which plays a key role in developing the adaptive estimator for the unknown parameters, including the high frequency gain. Compared with the state-of-art solutions in the literature [14][15], the proposed adaptive control scheme is of great importance due to the following distinctive features: i) the transient behaviour is significantly improved by avoiding any Nussbaum gain-like oscillation function; ii) the control gain function solves the singularity issue without imposing any projection operator on the parameter estimation, which means no lower or upper bound on any unknown parameter is required to be known prior; iii) No persistence excitation or sufficiently rich requirement is needed for the regressor and reference signals. Additionally, in spite of the relatively high dimension, the proposed is actually a rather simple algorithm to implement and tune, as the parameter estimators share the same one adaptation gain which ensures the stability with all positive values.

The remainder of the paper is organized as follows. Section II formulates the MRAC problem addressed. To help the reader capture the key ideas underlying the novel

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PIN scheme, in Section III, we presented a controller along with some adaptive laws that is easy to understand but characterized with some implementation issues. Then, an realizable PIN scheme is given in Section IV that solves the aforementioned issues and achieves the objective of asymptotic tracking. Section V illustrating the performance of the proposed PIN-MRAC scheme via a numerical example. Finally, the paper is wrapped-up with concluding remarks in Section VI.

In following sections, for the benefit of the reader, all the equations relevant to the implementation of the PIN adaptive controller will be marked by a gray background. In particular, we will highlight relevant formulas involving filtering of the regressor, parameter adaptation and definition of the control input.

Notation: Throughout this paper, we use the compact double-squared bracket notation to denote the image of an integrable signal $x(t)$ under the Laplace transform, $x(s) = [\![x(t)]\!] = \mathcal{L}\{x\}(s)$.

II. Problem Formulation

Consider a LTI SISO system described by

$$y(s) = [\![y(t)]\!] = b\frac{N(s)}{D(s)}[\![u(t)]\!], \tag{1}$$

where $u, y \in \mathbb{R}$ denote the plant input and output, respectively. The polynomials $N(s)$ and $D(s)$ are monic, coprime polynomials with unknown coefficients. The constant scalar parameter b, commonly referred to as "*high frequency gain*" is assumed *unknown* in this work. The following assumptions regarding the plant are formulated:

$(A.1)$ the degrees m and n of $N(s)$ and $D(s)$, respectively, are known and $\rho := n - m = 1$ (relative degree unity);

$(A.2)$ the polynomial $N(s)$ is Hurwitz.

Compared to assumptions made in the standard MRAC problem [16], [13], here we note by its absence is the assumption of prior knowledge of the high-frequency gain b. In this work, not only the sign of b is not needed, but also avoid any prior information on the lower or upper bounds of the value of b, which is usually needed in the methods [14][13] involving switching projection. Completely eliminating this assumption without resorting to any Nussbaum-like functions is the main subject of interest of this paper.

The objective of MRAC is to determine a bounded control input $u(t)$ using a differentiator-free controller such that the trajectories of the closed-loop system are bounded and the output $y(t)$ of the controlled plant tends asymptotically to the output $y_r(t)$ of the reference model

$$y_r(s) = \frac{1}{D_r(s)}[\![r(t)]\!], \tag{2}$$

where $r(t)$ is termed "reference command" and assumed to be a uniformly-bounded piece-wise continuous function of time, and $D_r(s)$ is a Hurwitz monic first-order polynomial having the form

$$D_r(s) = s + p$$

with $p > 0$. Without loss of generality, we have selected a first order reference model with no zeros, which is typically adopted in MRAC scheme. The controller presented in the sequel can be extend verbatim for a general SPR reference model.

Using the well-known results from linear adaptive control theory, we can express the tracking error

$$\tilde{y}(t) := y(t) - y_r(t)$$

in the so-called Elliot's parameterization [16] form

$$\dot{\tilde{y}} = -p\tilde{y} + b\left(-\xi^\top\theta + u\right), \quad \tilde{y}(0) = y(0) - y_r(0) \tag{3}$$

where $\theta \in \mathbb{R}^{2n+1}$ is a vector of unknown constant parameters and $\xi \in \mathbb{R}^{2n+1}$ is a vector of regressors obtained by collating the reference signal with filtered input and output signals reference by

$$\begin{aligned} \xi_u^\top(s) &= \frac{1}{L(s)}[1\ s\ \dots\ s^{n-1}(t)][\![u(t)]\!] \in \mathbb{R}^n, \\ \xi_y^\top(s) &= \frac{1}{L(s)}[1\ s\ \dots\ s^{n-1}(t)][\![y(t)]\!] \in \mathbb{R}^n, \\ \xi^\top(t) &= [\xi_u^\top(t)\ \xi_y^\top(t)\ r(t)] \in \mathbb{R}^{2n+1}, \end{aligned} \tag{4}$$

where $L(s)$ an arbitrary Hurwitz polynomial of order n given by:

$$L(s) = s^n + \alpha_{n-1}s^{n-1} + \dots + \alpha_1 s + \alpha_0.$$

Here, note that we have used the parametrization form of adaptive control due to Elliot, in which the regressors are obtained by filtering the input-output with filters of n-th order, in place of the more common parametrization that uses regressors obtained through filters of order $n-1$ plus a direct feedthrough from the plant output. The parametrization of Elliott is discussed in Paragraph 9.4.1 of [16] in the context of direct-adaptive pole placement and in Paragraph 10.6.1 of the same book in the design of adaptive controllers for multivariable systems. Notably, it does not introduce overparametrization compared to the conventinonal parametrization.

Elliott's parametrization of adaptive controllers represents the dual formulation of the more famous Kreisselmeier parametrization used to solve the adaptive observer problem [17]. A state-space realization of the filters (4) is given by the so-called K-filters (Kreisselmeier):

$$\begin{aligned} \dot{\xi}_u &= F\xi_u + gu, \quad \xi_u(0) \in \mathbb{R}^n \\ \dot{\xi}_y &= F\xi_y + gy, \quad \xi_y(0) \in \mathbb{R}^n \end{aligned}$$

where (F, g) is a pair of matrix verifying $L(s) = \det(sI - F)^{-1}g$. For instance, one may choose

$$F = \begin{bmatrix} -\alpha_{n-1} & -\alpha_{n-2} & -\alpha_{n-3} & \dots & -\alpha_0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}, g = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

in a controllable canonical form. In this work, the use of Elliot parametrization is not mandatory, one can also choose the usual form of the regressors vector of dimension $2n$, where $L(s)$ is a Hurwitz polynomial of degree $n-1$ and $\xi^\top(t) = [u(t)\ \xi_u^\top(t)\ y(t)\ \xi_y^\top(t)\ r(t)]$. In the forthcoming analysis, when clear from the context, the time arguments of function are omitted for brevity.

Posing $\vartheta := b\theta$ in (3), we get the tracking error equation

$$\dot{\tilde{y}} = -p\tilde{y} - \xi^\top\vartheta + bu. \tag{5}$$

Multiplying both sides of (5) by $\beta := b^{-1}$ to obtain the key tracking error equation

$$\beta\dot{\tilde{y}} = -p\beta\tilde{y} - \xi^\top\theta + u. \tag{6}$$

It is apparent from this equation that, compared to conventional adaptive control, the uncertainty now entering the left-hand-side through the unknown constant β affect in particular the tracking error derivative $\dot{\tilde{y}}$. The underlying idea of the proposed scheme is to inject, through the input, a signal that contains $\dot{\tilde{y}}$ in order to cancel this derivative-dependent uncertainty.

III. The PIN scheme: output error derivative injection through Parameter-dependent Input Normalization

The aim of this section is that of explaining the key logic underlying the proposed controller. To convey the main ideas of the PIN scheme, we first assume the output derivative $\dot{\tilde{y}}$ is available. Clearly, this is just a temporary relaxation of assumptions with the sole intention of making the reader approaching gradually the ultimate formulation.

Different from conventional adaptive laws, the proposed control input is constructed with a parameter-dependent normalization as follows

$$u = \frac{1}{1+\varkappa\hat{b}}\left(\xi^\top\hat{\theta} + \varkappa\xi^\top\hat{\vartheta}\right), \tag{7}$$

where $\hat{\theta}, \hat{\vartheta} \in \mathbb{R}^{2n+1}, \hat{b}, \hat{\beta} \in \mathbb{R}$ denote the estimates of the parameters θ, ϑ, b, β respectively, and $\varkappa : (t, \hat{\beta}, \hat{b}) \mapsto \mathbb{R}$ is a gain function of the estimated parameters that will be designed later. $\varkappa$ represents a key component of the scheme and is named as the *Derivative Injection Gain*. We will justify its name in the following lines. Indeed, by rearranging the expression (7) for the control law we get

$$\begin{aligned} u &= \pm p\varkappa\tilde{y} + \xi^\top\hat{\theta} + \varkappa\xi^\top\hat{\vartheta} - \varkappa\hat{b}u. \\ &= -p\varkappa\tilde{y} + \xi^\top\hat{\theta} - \varkappa\left(-p\tilde{y} - \xi^\top\hat{\vartheta} + \hat{b}u\right) \pm \varkappa\dot{\tilde{y}}. \end{aligned}$$

to which we added and subtracted terms $p\varkappa\tilde{y}$ and $\varkappa\dot{\tilde{y}}$ to the right-hand side of the equation, so that, by defining $\tilde{b} := \hat{b} - b$ and $\tilde{\vartheta} := \hat{\vartheta} - \vartheta$, together with (5), we can further write

$$\begin{aligned} u &= -p\varkappa\tilde{y} + \xi^\top\hat{\theta} - \varkappa\left(-\xi^\top\tilde{\vartheta} + \tilde{b}u\right) - \varkappa\dot{\tilde{y}} \\ &= -p\varkappa\tilde{y} + \xi^\top\hat{\theta} + \varkappa\xi^\top\tilde{\vartheta} - \varkappa\tilde{b}u - \varkappa\dot{\tilde{y}} \end{aligned} \tag{8}$$

where it stems clear that $\varkappa$ modulates the injection of the tracking error derivative in the control signal. Substituting (8) in (6), one has:

$$(\beta + \varkappa)\dot{\tilde{y}} = p\tilde{\beta}\tilde{y} - p(\hat{\beta} + \varkappa)\tilde{y} + \xi^\top\tilde{\theta} + \varkappa\xi^\top\tilde{\vartheta} - \varkappa\tilde{b}u$$

with $\tilde{\beta} := \hat{\beta} - \beta$ and $\tilde{\theta} := \hat{\theta} - \theta$. Moreover, using again the relation that $\beta = \hat{\beta} - \tilde{\beta}$ to obtain the key expression:

$$(\hat{\beta} + \varkappa)\dot{\tilde{y}} = \tilde{\beta}\dot{\tilde{y}} - p(\hat{\beta} + \varkappa)\tilde{y} + p\tilde{\beta}\tilde{y} + \xi^\top\tilde{\theta} + \varkappa\xi^\top\tilde{\vartheta} - \varkappa\tilde{b}u$$

and finally

$$\dot{\tilde{y}} = -p\tilde{y} + \frac{\dot{\tilde{y}} + p\tilde{y}}{\hat{\beta} + \varkappa}\tilde{\beta} + \frac{1}{\hat{\beta} + \varkappa}\xi^\top\tilde{\theta} + \frac{\varkappa}{\hat{\beta} + \varkappa}\xi^\top\tilde{\vartheta} - \frac{\varkappa u}{\hat{\beta} + \varkappa}\tilde{b}. \tag{9}$$

The above expression is a standard error equation which calls for the following adaptation laws

$$\begin{aligned} \dot{\hat{\beta}} &= -\frac{\mu}{\hat{\beta} + \varkappa}\left(\dot{\tilde{y}} + p\tilde{y}\right)\tilde{y} \\ \dot{\hat{\theta}} &= -\frac{\mu}{\hat{\beta} + \varkappa}\xi\tilde{y} \\ \dot{\hat{\vartheta}} &= -\frac{\mu}{\hat{\beta} + \varkappa}\varkappa\xi\tilde{y} \\ \dot{\hat{b}} &= \frac{\mu}{\hat{\beta} + \varkappa}\varkappa u\tilde{y} \end{aligned} \tag{10}$$

with μ is a positive adaptation gain. Following standard SPR-Lyapunov arguments in [13, Chapter4] and considering the following Lyapunov function

$$V(t) = \frac{\tilde{y}^2}{2} + \frac{\tilde{\theta}^\top\tilde{\theta}}{2\mu} + \frac{\tilde{\vartheta}^\top\tilde{\vartheta}}{2\mu} + \frac{\tilde{b}^2 + \tilde{\beta}^2}{2\mu}$$

it is immediate to see that adaptive law (10) for (9) guarantees that $\tilde{y} \to 0$ as t goes to infinity. The detailed proof is omitted here due to space limitation.

We underscore the fact that, to have an implementable control of (8) and non-singular adaptation laws of (10), we need to design the derivative injection gain $\varkappa$ to ensure that both $1 + \varkappa\hat{b} \neq 0$ and $\hat{\beta} + \varkappa \neq 0$, for any possible real value of $\hat{b}$ and $\hat{\beta}$. By choosing the following expression for the derivative injection gain $\varkappa$, which is discontinuous in $\hat{\beta}$:

$$\varkappa = \begin{cases} -\left(|\hat{\beta}| + \underline{\varkappa}\right), & \hat{b} < 0 \\ \left(|\hat{\beta}| + \underline{\varkappa}\right), & \hat{b} \geq 0 \end{cases}$$

with $\underline{\varkappa} > 0$ arbitrary, we achieve both tasks.

While by the aforementioned formulation (9), the mechanism proposed in this section that permits to realize the output-derivative-injection can be easily understood, it has two main drawbacks:

1) in this setup, we cannot guarantee a finite dwell-time for the discontinuous output-injection gain $\varkappa$ since we cannot exclude that $\hat{b}$ exhibits high-frequency zero-crossings.
2) Noting that the adaptation law for $\dot{\hat{\beta}}$ contains the term $\dot{\tilde{y}}$, it stems clear that this formulation cannot be trivially extended to the more common case where $\dot{\tilde{y}}$ is unavailable.

We will overcome both these issues in next section by modifying the PIN control input with a parameter-derivative injection and by assigning to $\varkappa$ a hysteretic behavior.

IV. Realizable PIN scheme through parameter derivative injection and hysteresis

Injecting the parameter derivatives through the control input, weighted by filtered regressors, is a typical provision adopted to implement adaptive schemes for relative degree-two system, transforming them into equivalent relative-degree one system that can be tackled by standard methods. The interested reader can refer to Paragraph 5.4.3 of [16] for an insight on such a methodology. Here, we will resort to an analogous procedure to overcome the unavailability of $\dot{\tilde{y}}$. Let us introduce a vector of extended parameter vector

$$\phi^\top := [\beta, \theta^\top, \vartheta^\top, b] \in \mathbb{R}^{4n+4}$$

and a vector of filtered regressors

$$\omega^\top := [\omega_\beta, \omega_\theta^\top, \omega_\vartheta^\top, \omega_b] \in \mathbb{R}^{4n+4} \tag{11}$$

where each component is generated by filtering the regressor signals appearing in (9) as

$$\dot{\omega}_\beta \quad = \quad -p\omega_\beta + \frac{\dot{\tilde{y}} + p\tilde{y}}{\hat{\beta} + \varkappa}, \quad \omega_\beta(0) \in \mathbb{R} \tag{12}$$

$$\begin{aligned} \dot{\omega}_\theta \quad &= \quad -p\omega_\theta + \frac{\xi^\top}{\hat{\beta} + \varkappa}, \quad \omega_\theta(0) \in \mathbb{R}^{2n+1} \\ \dot{\omega}_\vartheta \quad &= \quad -p\omega_\vartheta + \frac{\varkappa\xi^\top}{\hat{\beta} + \varkappa}, \quad \omega_\vartheta(0) \in \mathbb{R}^{2n+1} \\ \dot{\omega}_b \quad &= \quad -p\omega_b - \frac{\varkappa u}{\hat{\beta} + \varkappa}, \quad \omega_b(0) \in \mathbb{R}. \end{aligned} \tag{13}$$

Note that, while we have marked in gray the filtered dynamics (13), since these filters represent the final machinery to obtain the correspondent filtered regressors, we have not marked (12). Indeed, an easier to implement expression for ω_β, not involving the differentiation of $\dot{\tilde{y}}$, will be given later in (19).

Now, denoting the estimates of ϕ by the parameter vector $\hat{\phi}^\top := [\hat{\beta}, \hat{\theta}^\top, \hat{\vartheta}^\top, \hat{b}]$, we propose a realizable PIN control input takes the form of

$$u = \frac{1}{1+\varkappa\hat{b}}\Big(\xi^\top\hat{\theta} + \varkappa\xi^\top\hat{\vartheta} + (\hat{\beta} + \varkappa)\omega^\top\dot{\hat{\phi}}\Big) \tag{14}$$

where, compared to (7), we have introduced the additional term $(\hat{\beta} + \varkappa)\omega^\top\dot{\hat{\phi}}$ containing the derivative of the estimation parameter vector $\hat{\phi}^\top$ whose adaptive law will be given later by (20) and (22). Again, following the similar procedure in (8), we can rewrite (14) as

$$u = -p\varkappa\tilde{y} + \xi^\top\hat{\theta} + \varkappa\xi^\top\hat{\vartheta} - \varkappa\tilde{b}u - \varkappa\dot{\tilde{y}} + (\hat{\beta} + \varkappa)\omega^\top\dot{\hat{\phi}} \tag{15}$$

Plugging (14) into the error equation (6) we obtain

$$\begin{aligned} \dot{\tilde{y}} &= -p\tilde{y} + \frac{\dot{\tilde{y}} + p\tilde{y}}{\hat{\beta}+\varkappa}\tilde{\beta} + \frac{1}{\hat{\beta}+\varkappa}\xi^\top\tilde{\theta} + \frac{\varkappa}{\hat{\beta}+\varkappa}\xi^\top\tilde{\vartheta} - \frac{\varkappa u}{\hat{\beta}+\varkappa}\tilde{b} + \omega^\top\dot{\hat{\phi}} \\ &= \dot{\omega}^\top\tilde{\phi} + \omega^\top\dot{\hat{\phi}} \end{aligned} \tag{16}$$

where we have used the notation $\tilde{\phi} := \hat{\phi} - \phi$ for the extended parameter error vector, yielding to the algebraic linear-in-the parameter expression

$$\tilde{y} = \omega^\top\tilde{\phi}, \tag{17}$$

which is central in deriving the parameter adaptation law for the system.

Next, we will exploit the freedom in designing the injection term $\varkappa$ to enforce finite dwell-time guarantees on its discontinuous dynamics, relying on a hysteretic switching logic. It will turn out that $\varkappa$ will be left-discontinuous in time exhibiting jumps of finite amplitude. In this connection, let us make the following choice for $\varkappa$:

$$\varkappa = \hat{b} + \varkappa_h$$

with $\varkappa_h$ to be designed. With this choice, to avoid singularities in the PIN control law and in the right-hand sides of (13), we need to choose $\varkappa_h$ such that

$$1 + \varkappa\hat{b} = 1 + \hat{b}^2 + \hat{b}\varkappa_h \neq 0,$$

and

$$\hat{\beta} + \varkappa = \hat{\beta} + \hat{b} + \varkappa_h \neq 0,$$

for any possible value of $\hat{b}, \hat{\beta}$. Now, for any $\varkappa_h : |\varkappa_h| \leq 1$ it holds that $1 + \hat{b}^2 + \hat{b}\varkappa_h \geq \frac{3}{4}$, for any possible real value of $\hat{b}$. Moreover, let us assign the following hysteretic switching dynamics to $\varkappa_h$:

$$\varkappa_h(0) = \begin{cases} -1, & \hat{\beta}(0) + \hat{b}(0) < \frac{1}{2} \\ 1, & \hat{\beta}(0) + \hat{b}(0) \geq \frac{1}{2} \end{cases}$$

$$\varkappa_h(t) = \begin{cases} -1, & \hat{\beta}(t) + \hat{b}(t) \leq -\frac{1}{2} \\ 1, & \hat{\beta}(t) + \hat{b}(t) \geq \frac{1}{2} \\ \varkappa_h(t^-), & -\frac{1}{2} < \hat{\beta}(t) + \hat{b}(t) < \frac{1}{2} \end{cases} \quad , t > 0$$

where $\varkappa_h(t^-) := \lim_{\tau \to t^-} \varkappa_h(\tau)$ denotes the left-hand limit of the function $\varkappa_h$ at time t. Thanks to the above choice, we have that $|\hat{\beta} + \hat{b} + \varkappa_h| \geq 1/2$, for any real $\hat{\beta}, \hat{b}$.

Having assigned a specific formulation to $\varkappa$, we can now proceed by clarifying how it is possible to overcome the unavailability of $\dot{\tilde{y}}$ in the right-hand side of (12). Indeed, we will show that all the elements of the filtered regressors vector ω are computable from known quantities without direct differentiation. While this is obvious for $\omega_\theta, \omega_\vartheta$ and ω_b, some additional algebra is needed to show that also ω_β can be obtained without differentiation. In particular, we show that the evolution of $\omega_\beta(t)$ can be obtained through causal filters plus with a direct-feedthrough term from $\tilde{y}$. Referring to the differential equation of ω_β in (12), it holds

$$\omega_\beta(t) = \int_0^t e^{-p(t-\tau)} \frac{p\tilde{y}(\tau) + \dot{\tilde{y}}(\tau)}{\hat{\beta}(\tau) + \varkappa(\tau)} d\tau$$

$$= \int_0^t \frac{e^{-p(t-\tau)} p\tilde{y}(\tau)}{\hat{\beta}(\tau) + \varkappa(\tau)} d\tau + \frac{\tilde{y}(t)}{\hat{\beta}(t) + \varkappa(t)} - \frac{\tilde{y}(0)}{\hat{\beta}(0) + \varkappa(0)} e^{-pt}$$

$$- \int_0^t e^{-p(t-\tau)} \left(\frac{p}{\hat{\beta}(\tau) + \varkappa(\tau)} - \frac{\dot{\hat{\beta}}(\tau) + \frac{d}{d\tau}\varkappa(\tau)}{\left(\hat{\beta}(\tau) + \varkappa(\tau)\right)^2} \right) \tilde{y}(\tau) d\tau$$

where we have used the formula of the integration by parts. Recalling the relation $\varkappa = \hat{b} + \varkappa_h$, it follows that

$$\omega_\beta(t) = \frac{\tilde{y}(t)}{\hat{\beta}(t) + \varkappa(t)} - \frac{\tilde{y}(0)}{\hat{\beta}(0) + \varkappa(0)} e^{-pt}$$

$$+ \int_0^t e^{-p(t-\tau)} \left(\frac{\dot{\hat{\beta}}(\tau) + \dot{\hat{b}}(\tau) + \frac{d}{d\tau}\varkappa_h(\tau)}{\left(\hat{\beta}(\tau) + \varkappa(\tau)\right)^2} \right) \tilde{y}(\tau) d\tau.$$

Note that the term involving the time derivative of $\varkappa_h$ in above equation can be further written as

$$\int_0^t e^{-p(t-\tau)} \frac{\frac{d}{d\tau}\varkappa_h(\tau)}{\left(\hat{\beta}(\tau) + \varkappa(\tau)\right)^2} \tilde{y}(\tau) d\tau$$

$$= \sum_{t_\delta \in T_\Delta} \int_0^t e^{-p(t-\tau)} \left(\frac{2\sigma_h(t_\delta)\delta(\tau - t_\delta)}{\left(\hat{\beta}(\tau) + \varkappa(\tau)\right)^2} \right) \tilde{y}(\tau) d\tau$$

$$= \sum_{t_\delta \in T_\Delta} e^{-p(t-t_\delta)} \frac{2\sigma_h(t_\delta)}{\left(\hat{\beta}((t_\delta)) + \varkappa((t_\delta))\right)^2} \tilde{y}(t_\delta) \tag{18}$$

where $\delta(\cdot)$ is the Dirac's delta operator, $T_\Delta := \{t \in \mathbb{R}_{\geq 0} : \varkappa_h(t) \neq \varkappa_h(t^-)\}$ is the set containing all the time instants in which $\varkappa_h(t)$ exhibits a jump, while $\sigma_h(t) := \mathrm{sign}(\varkappa_h(t) - \varkappa_h(t^{-1}))$ is a left-discontinuous function that encodes the up-fronts or down-fronts of $\varkappa_h$. Substituting (18) into $\omega_\beta(t)$ and neglecting the exponentially fading term $-\frac{\tilde{y}(0)}{\hat{\beta}(0)+\varkappa(0)} e^{-pt}$, the signal $\omega_\beta(t)$ takes the form

$$\omega_\beta(t) = \frac{\tilde{y}(t)}{\hat{\beta}(t) + \varkappa(t)} + \int_0^t e^{-p(t-\tau)} \frac{\dot{\hat{\beta}}(\tau) + \dot{\hat{b}}(\tau)}{\left(\hat{\beta}(\tau) + \varkappa(\tau)\right)^2} \tilde{y}(\tau) d\tau$$

$$+ \sum_{t_\delta \in T_\Delta} e^{-p(t-t_\delta)} \left(\frac{2\sigma_h(t_\delta)}{\left(\hat{\beta}(t_\delta) + \varkappa(t_\delta)\right)^2} \right) \tilde{y}(t_\delta).$$

More conveniently, it is implementable by the formula

$$\omega_\beta(t) = \frac{\tilde{y}(t)}{\hat{\beta}(t)+\varkappa(t)} + \omega_\beta^{'}(t) + \omega_\beta^{''}(t), \tag{19}$$

where $\omega_\beta^{'}(t)$ is given by the linear filter

$$\dot{\omega}_\beta^{'} = -p\omega_\beta^{'} + \frac{\dot{\hat{\beta}}(\tau)+\dot{\hat{b}}(\tau)}{\left(\hat{\beta}(\tau)+\varkappa(\tau)\right)^2} \tilde{y}(\tau),$$

with $\omega_\beta^{'}(0) = 0$, while $\omega_\beta^{''}(t)$ can be obtained by the left-discontinuous dynamics

$$\begin{aligned} \dot{\omega}_\beta^{''}(t) &= -p\omega_\beta^{''}(t), & t \notin T_\Delta, \\ \omega_\beta^{''}(t) &= \omega_\beta^{''}(t^-) + \left(\frac{2\sigma_h(t))}{\left(\hat{\beta}(t))+\varkappa(t))\right)^2} \right) \tilde{y}(t), & t \in T_\Delta, \end{aligned}$$

with $\omega_\beta^{''}(0) = 0$. Note that from a practical standpoint, in order to detect whether the current t belongs to T_Δ it is sufficient to check, if $\varkappa_h$, which is an available synthetic signal, exhibits a jump between -1 to 1 or vice-versa. Moreover, the hysteretic dynamics assigned to $\varkappa_h$ ensures that two jumps have a minimum finite time-separation.

After previous derivations, we are now in position to construct the adaptation law for the parameter estimates $\hat{\phi}$. The linear-in-the parameters expression (17) may, in principle, suggest to adopt the following conventional normalized adaptive law for relative-degree-0 systems:

$$\dot{\hat{\phi}} = -\mu \frac{\omega}{1 + \omega^\top \omega} \tilde{y},$$

with $\mu > 0$ a user-defined scalar constant used to tune the speed of adaptation. However, due to the jump discontinuities of ω, the use of such an adaptation mechanism does not fit the requirement of uniform continuity needed in the adaptive context to apply the Barbălat Lemma or Lyapunov-Like stability results. To overcome this issue and streamline the proof of stability let us introduce a further filter

$$\dot{\tilde{y}}_f = -p\tilde{y}_f + \frac{1}{1+\omega^\top \omega} \tilde{y}, \tag{20}$$

with initial condition $\tilde{y}_f(0) = \tilde{y}(0)$. In view of (17), the right-hand-side of (20) can be expressed in terms of the parameter error

$$\dot{\tilde{y}}_f = -p\tilde{y}_f + \frac{1}{1 + \omega^\top \omega} \omega^\top \tilde{\phi}. \tag{21}$$

This relative-degree-1 dynamics calls for the following adaptive law

$$\dot{\hat{\phi}} = -\mu \frac{\omega}{1+\omega^\top \omega} \tilde{y}_f. \tag{22}$$

where normalization, not required in a conventional relative-degree-1 setting, is needed here to ensure the boundedness of the normalized regressors, to allow later on the application of the Song-and-Tao's higher-order asymptotic stability result [18]. In the sequel, the stability analysis of the proposed scheme will be performed using the following well-known result, that descends from Barbălat's Lemma:

Lemma 4.1: (Lyapunov-Like Lemma [19]) If a scalar function $V(t,x)$ satisfies the following conditions:

- $V(t,x)$ is lower bounded
- $\dot{V}(t,x)$ is semi-negative definite
- $\dot{V}(t,x)$ is uniformly continuous in time

then $\dot{V}(t,x) \xrightarrow[t\to\infty]{} 0$.

Choosing the candidate Lyapunov function $V = \frac{1}{2}\left(\tilde{y}_f^2 + \frac{1}{\mu}\tilde{\phi}^\top \tilde{\phi}\right)$, after trivial algebra we obtain

$$\dot{V} = -p\tilde{y}_f^2 \ < 0.$$

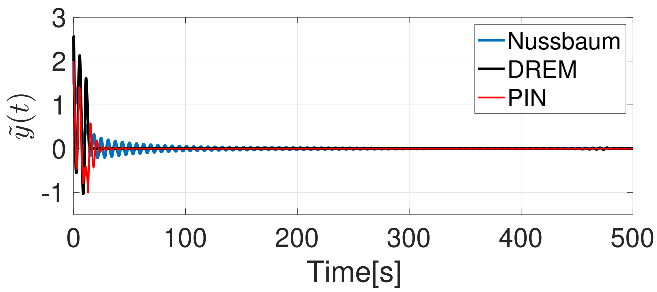

Fig. 1. Comparison of the time behaviour of tracking error.

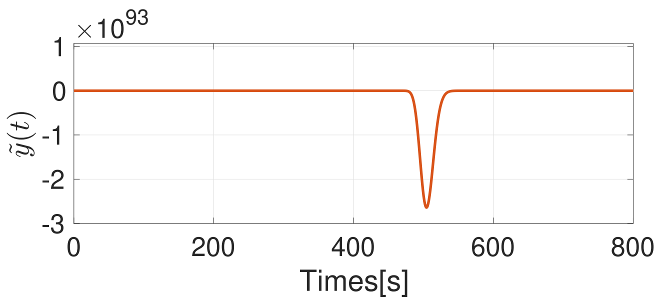

Fig. 2. Time behaviour of the tracking error $\tilde{y}(t)$ of Nussbaum-gain-based method[16].

Since $\tilde{y}_f$ is uniformly continuous, in view of the Lyapunov-Like Lemma it can be proven that $\tilde{y}_f \underset{t\to\infty}{\longrightarrow} 0$. A further step is needed to prove that $\tilde{y} \underset{t\to\infty}{\longrightarrow} 0$. Very recently, Song and Tao in [18] and [20] have shown that by using conventional MRAC schemes it possible to extend the global convergence property to higher-order derivatives of the tracking error. In particular Corollary 3.1 of [18] can be invoked to establish that, being (21) relative-degree-unity, the parameters bounded as well as the normalized regressor vector $\frac{1}{1+\omega^\top\omega}\omega^\top$, it holds that

$$\dot{\tilde{y}}_f \underset{t\to\infty}{\longrightarrow} 0 \implies \tilde{y} \underset{t\to\infty}{\longrightarrow} 0$$

which indicates the control objective of MRAC problem is achieved.

V. Numerical Example

In this section, we present simulation results to illustrate the effectiveness of the proposed PIN MRAC method.The

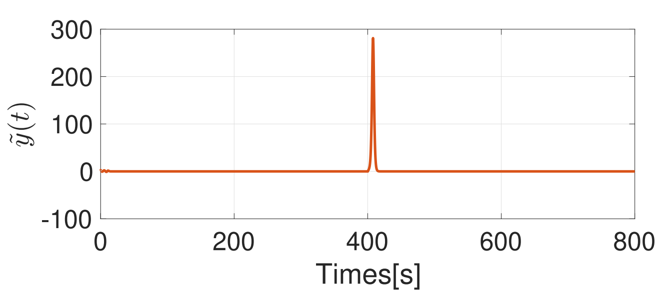

Fig. 3. Time behaviour of the tracking error $\tilde{y}(t)$ of DREM-based MRAC[14].

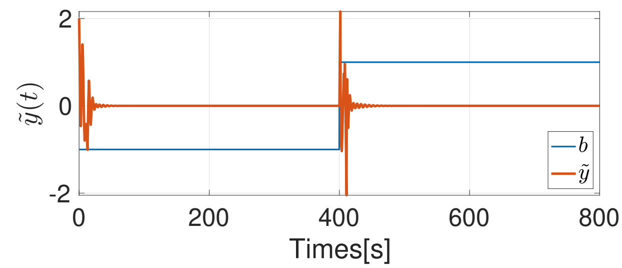

Fig. 4. Time behaviour of the tracking error $\tilde{y}(t)$ of the PIN-MRAC.

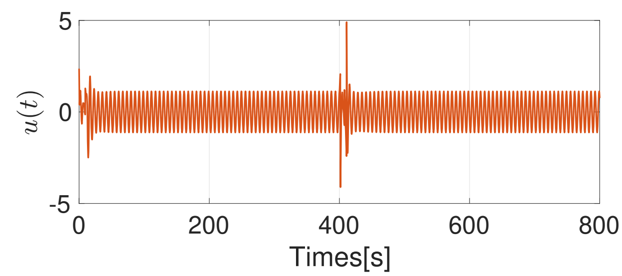

Fig. 5. Time behaviour of the control input $u(t)$ of the PIN-MRAC.

proposed algorithm is compared with a classical scheme using a Nussbaum-gain [16, Chp 9] and a recently developed DREM-based technique[14]. The Runge-Kutta integration method has been employed for all simulations with fixed sampling interval $T_s = 10^{-3}s$. Consider a stable second-order system

$$[\![y(t)]\!] = b\frac{s+b_0}{s^2+a_1s+a_0}[\![u(t)]\!]$$

with unknown parameters $b_0 = 1$, $a_1 = 2.1$, $a_0 = 0.2$ and $b = -1$. The reference model is given by

$$y_m(t) = \frac{1}{s+1}r(t)$$

that is $p = 1$, and the reference signal $r(t) = \sin t$ to be tracked is not sufficiently rich.

The initial conditions of the plant and reference model are taken as $y(0) = 2$ and $y_r(0) = 0$, leads to $\tilde{y}_f(0) = 2$. The regressor signal given by (4), (13) and (19) are all initialized with zeros. The filter $L(s)$ is selected as $L(s) = s^2+3s+2$. The tuning gains are chosen as $\mu = 1$. By assigning $\hat{\phi}^\top(0) = [-1\,,\, 0_{1\times 10}\,,\, -1]$, we underscore the fact this is indeed a stringent test, as the simulation is performed taking into account for the wrong initial guess of control direction. For the fair of comparison, the tuning parameters($\gamma_{j0} = 1$ for $j = 1, 2...5$, $\underline{k}_p = 0.1$) and initial conditions $\hat{\theta}(0) = [-10, 10, -5, 5]$ of the other two methods are selected after a process of trial-and-error to achieve similar transient and steady state behaviour.

As shown in Fig.1, Nussbaum-gain-based MRAC has a slight slower converge speed, but all three methods succeed in tracking the output of the reference model given a wrong initial guess of the control direction. Using exactly same set of initial conditions and tuning parameter, we now consider a more challenging case in which the high frequency gain b features an abrupt change of sign at 400s.

$$b = \begin{cases} -1 & \text{if } t \in [0, 400) \\ 1 & \text{if } t \in [400, 800] \end{cases}$$

The obtained simulation results are shown in Fig. 2-5, respectively for tracking errors $\tilde{y}$ of three methods and control signal $u(t)$. We draw to the readers' attention the difference in scales of the time plot of $\tilde{y}$ of three methods, both in time and in amplitude. It can be easily seen that PIN-MRAC achieves the control objectives with superior transient behaviour. This suggests that the proposed controller does not have the practical admissible concerns which usually shared by the Nussbaum-gain-based methods [6][11] and the DREM-based techniques [12][14].

VI. Concluding Remarks

The model reference trajectory tracking problem for an uncertain relative-degree-unity system is addressed in this paper by means of an original PIN-based adaptive controller, which allows us to completely remove the bottleneck assumption on the prior knowledge of the high frequency gain. Other novelties of the proposed scheme lie in the facts that it does not use any oscillating gain function that may periodical destabilizing the system nor employ a projection operation to ensure the non-singularity of the control gain. Moreover, no persistent excitation requirement is needed for the convergence of the output error. Simulation results are consistent with the theoretical analysis and reveal superior transient behaviour of the PIN controller. Current research is under way to apply the similar construction to the systems with relative degree greater than one. The second attractive future research direction is the extension of current results to the MIMO cases, where the removal of assumption on the high frequency gain matrix is more challenge and far from solved. In addition, to further broaden the applicability of the proposed techniques in practical settings, the assessment of the robustness properties of the proposed PIN algorithm is of interest too.

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